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Exam I: MTH 221, Spring 2016

Ayman Badawi

QUESTION 1. (i) Given $A^{-1} = \begin{bmatrix} 2 & 4 & 6 & 9 \\ -2 & -2 & 4 & 1 \\ 0 & -2 & -8 & 11 \\ -2 & -4 & -6 & -7 \end{bmatrix}$. Then $|2A| =$

a. 256 b. 32 c. $\frac{1}{8}$ ~~d.~~ 1

(ii) Given $A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & -1 & 4 \end{bmatrix}$. The solution set to the system $AX = \begin{bmatrix} 10 \\ 0 \\ -6 \\ 6 \end{bmatrix}$ is

a. ϕ (empty set)
b. $\{(10 + 2x_4,\ 6 - 4x_4,\ -6 + 4x_4, x_4) \mid x_4 \in R\}$
c. $\{(4 + 2x_4,\ -4x_4,\ -6 + 4x_4, x_4) \mid x_4 \in R\}$
~~d.~~ $\{(4 + 2x_4,\ 6 - 4x_4,\ -6 + 4x_4, x_4) \mid x_4 \in R\}$

(iii) Given $A = \begin{bmatrix} a & -3 & 2 \\ b & 1 & 1 \\ c & 0 & -3 \end{bmatrix}$ such that $|A| = 3$, for some fixed numbers a, b, c. Then $(1, 2)$-entry of A^{-1} is

~~a.~~ -3 b. 3 c. $\frac{3b+c}{3}$ d. $\frac{-3b-c}{3}$ e. $\frac{-7}{3}$

(iv) Let $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{bmatrix}$. Then $A^{-1} =$

a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ ~~d.~~ $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

(v) All possible values of a, b, c that make the matrix $A = \begin{bmatrix} 1 & 4 & -6 \\ -1 & a & b \\ -1 & -4 & c \end{bmatrix}$ equivalent (row-equivalent) to I_3 are:

a. $a \neq 0, b \neq 0$, and $c \neq 0$
b. $a \neq 0$, $c \neq 0$, and b any real number
~~c.~~ $a \neq -4$, $c \neq 6$, and b any real number
d. $a = 1$, $c = 1$ and $b = 0$
e. $a = 1$, $c = 1$ and b any real number.

(vi) Let $A = \begin{bmatrix} a & 5 & 7 \\ b & 8 & 6 \\ c & 5 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} a+4 & 5 & 7 \\ b & 8 & 6 \\ c & 5 & 5 \end{bmatrix}$. Given $|A| = 30$. Then $|B| =$ (Hint: you may need to use the first row to find $|A|$ and $|B|$... then stare!)

a. 34 b. 26 c. 30 ~~d.~~ 70 e. 40

(vii) Let A be a 2×3 matrix such that $\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} A + 2A = \begin{bmatrix} 2 & 4 & 0 \\ 4 & 6 & 2 \end{bmatrix}$. Then $A =$

~~a.~~ $\begin{bmatrix} -1 & -1 & -1 \\ 3 & 5 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 3 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ c. $\begin{bmatrix} -2 & -2 & -2 \\ 6 & 10 & 2 \end{bmatrix}$ d. $\begin{bmatrix} 6 & 10 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

(viii) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$. Write $A = LU$, where L is lower triangular and U is upper triangular. Then

a. $L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. ~~b.~~ $L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$. c. $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. d. $L = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$.

(ix) Let $A = \begin{bmatrix} \sqrt{2} & 0.45 & 2 \\ -7 & 34 & -6 \\ 23 & 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 & 0 \\ -4 & 3 & 0 \\ 9 & 0 & -2 \end{bmatrix}$. Let $D = AB$. Then the third column of D is

a. $\begin{bmatrix} 30 \\ -26 \\ 12 \end{bmatrix}$ b. $\begin{bmatrix} -26 \\ 30 \\ 12 \end{bmatrix}$ ~~c.~~ $\begin{bmatrix} -4 \\ 12 \\ -6 \end{bmatrix}$ d. Not wasting my time to do messy calculation

(x) Given $A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ a & b & c \end{bmatrix}$, where a, b, c are some fixed real numbers. The solution set to the system $A^T X = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ is

~~a.~~ $\{(2, 3, 4)\}$ b. $\{(2, 3, a + b)\}$ c. $\{(a + b, 3, 2)\}$ d. $\{(a + 1, b + 2, c + 3)\}$

(xi) Given A is a 3×2 matrix and $A \xrightarrow{-3R_1 + R_2 \to R_2} B \xrightarrow{2R_2} D = \begin{bmatrix} 1 & 3 \\ -1 & 8 \\ 1 & 4 \end{bmatrix}$. Let F, W be two elementary matrices such that $FWA = D$. Then

a. $F = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $W = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$,

b. $F = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$, $W = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

~~c.~~ $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $W = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d. $F = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(xii) Given $A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. The solution set to the system $AX = \begin{bmatrix} 10 \\ 0 \\ -6 \\ 4 \end{bmatrix}$ is

a. $\{(10, 0, -6, 4)\}$ b. $\{(-18, 10, 10, 4)\}$ c. $\{(0, 2, -2, 4)\}$ ~~d.~~ $\{(12, -10, 10, 4)\}$

(xiii) Given A is a 5×5 matrix such that $A \xrightarrow{2R_1} A_1 \xrightarrow{R_2 + R_3 \to R_3} A_2 \xrightarrow{R_4 \leftrightarrow R_5} I_5$. Then the solution set to the system of linear equations $AX = \begin{bmatrix} 0.5 \\ -3 \\ 4 \\ -1 \\ 1 \end{bmatrix}$:

a. $\{(1, 1, 1, 1, 1)\}$ b. $\{(1, 1, 1, -1, 1)\}$ ~~c.~~ $\{(1, -3, 1, 1, -1)\}$ d. $\{(1, -3, 1, -1, 1)\}$

(xiv) Let A be a 2×2 matrix such that $A \xrightarrow{0.5R_1} A_1 \xrightarrow{4R_2 + R_1 \to R_1} B$. Let F, W be elementary matrices such that $FWB = A$. Then

a. $W = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$ and $F = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

~~b.~~ $W = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$ and $F = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

c. $W = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$ and $F = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$

d. $W = \begin{bmatrix} 1 & 0.25 \\ 0 & 1 \end{bmatrix}$ and $F = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

e. $W = \begin{bmatrix} 1 & -0.25 \\ 0 & 1 \end{bmatrix}$ and $F = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(xv) The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 7 & -1 & c \\ -1 & c & 1 & -c-10 \\ -2 & -13 & 2 & -c \end{bmatrix}$, where c is a real number. For what values of c will the system be consistent?

a. c can be any real number except -7.

b. c can be any real number except -7 and 0

~~c.~~ c must be either -2 or -5

d. c can be any real number except 7.

e. There are no values for c since the system is always inconsistent

(xvi) The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 1 & -1 & c \\ -1 & -1 & 2 & 8-c \\ 1 & 1 & 3 & 0 \end{bmatrix}$. Assume that the system is consistent for a fixed real number c. If $x_1 = -21$, then the value of x_2 is

~~a.~~ -3 b. 21 c. -61 d. There are infinitely many possible values for x_2.

(xvii) Let $A = \begin{bmatrix} 1 & a & 0 \\ -1 & 0 & b \\ 0 & -2a & 4 \end{bmatrix}$. For what values of a, b will the system $A^T X = \begin{bmatrix} 0.33 \\ 0.75 \\ 12.25 \end{bmatrix}$ have a unique solution?

a. $a \neq 0$ and $b \neq -4$

b. a any real number and $b \neq -2$

c. a any real number and $b \neq -4$

~~d.~~ $a \neq 0$ and $b \neq -2$

e. $a \neq 0$ and b any real number

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com